A Conjugate Gradient Algorithm For Analysis Of Variance

Conjugate Gradient Algorithms in Nonconvex OptimizationSpringer Science & Business Media

Shortly after the end of World War II high-speed digital computing machines were being developed. It was clear that the mathematical aspects of com putation needed to be reexamined in order to make efficient use of high-speed digital computers for mathematical computations. Accordingly, under the leadership of Min a Rees, John Curtiss, and others, an Institute for Numerical Analysis was set up at the University of California at Los Angeles under the sponsorship of the National Bureau of Standards. A similar institute was formed at the National Bureau of Standards in Washington, D. C. In 1949 J. Barkeley Rosser became Director of the group at UCLA for a period of two years. During this period we organized a seminar on the study of solu tions of simultaneous linear equations and on the determination of eigen values. G. Forsythe, W. Karush, C. Lanczos, T. Motzkin, L. J. Paige, and others attended this seminar. We discovered, for example, that even Gaus sian elimination was not well understood from a machine point of view and that no effective machine oriented elimination algorithm had been developed. During this period Lanczos developed his three-term relationship and I had the good fortune of suggesting the method of conjugate gradients. We dis covered afterward that the basic ideas underlying the two procedures are essentially the same. The concept of conjugacy was not new to me. In a joint paper with G. D.

The position taken in this collection of pedagogically written essays is that conjugate gradient algorithms and finite element methods complement each other extremely well. Via their combinations practitioners have been able to solve complicated, direct and inverse, multidemensional problems modeled by ordinary or partial differential equations and inequalities, not necessarily linear, optimal control and optimal design being part of these problems. The aim of this book is to present both methods in the context of complicated problems modeled by linear and nonlinear partial differential equations, to provide an in-depth discussion on their implementation aspects. The authors show that conjugate gradient methods and finite element methods apply to the solution of real-life problems. They address graduate students as well as experts in scientific computing.

Nonlinear conjugate gradient methods (CG) are especially efficient methods for the solution of the large scale unconstrained optimization problems, as these methods are characterized by low memory requirement and strong local and global convergence properties. However, line search in CG methods is of critical importance. Reducing line search cost in conjugate gradient algorithm is the main objective of this monograph. Based on the type of the line searches applied in the CG method, these methods are generally divided into three approaches. The conjugate gradient methods with exact line search, inexact line search and conjugate gradient without any line search. This study is intended to compare and analyze the efficiency and effectiveness of these optimization techniques through numerical simulations.

Two approaches are known for solving large-scale unconstrained optimization problems—the limited-memory quasi-Newton method (truncated Newton method) and the conjugate gradient method. This is the first book to detail conjugate gradient methods, showing their properties and convergence characteristics as well as their performance in solving large-scale unconstrained optimization problems and applications. Comparisons to the limited-memory and truncated Newton methods are also discussed. Topics studied in detail include: linear conjugate gradient methods, standard conjugate gradient methods, acceleration of conjugate gradient methods, hybrid, modifications of the standard scheme, memoryless BFGS preconditioned, and three-term. Other conjugate gradient methods with clustering the eigenvalues or with the minimization of the condition number of the iteration matrix, are also treated. For each method, the convergence analysis, the computational performances and the comparisons versus other conjugate gradient methods are given. The theory behind the conjugate gradient algorithms presented as a methodology is developed with a clear, rigorous, and friendly exposition; the reader will gain an understanding of their properties and their convergence and will learn to develop and prove the convergence of his/her own methods. Numerous numerical studies are supplied with comparisons and comments on the behavior of conjugate gradient algorithms for solving a collection of 800 unconstrained optimization problems of different structures and complexities with the number of variables in the range [1000,10000]. The book is addressed to all those interested in developing and using new advanced techniques for solving unconstrained optimization complex problems. Mathematical programming researchers, theoreticians and practitioners in operations research, practitioners in engineering and industry researchers, as well as graduate students in mathematics, Ph.D. and master students in mathematical programming, will find plenty of information and practical applications for solving large-scale unconstrained optimization problems and applications by conjugate gradient methods.

This paper develops the original conjugate gradient method and the idea of preconditioning a system. I also propose a unique type of additive-Schwarz preconditioner that can be solved in parallel, which creates a speed increase for large systems. To show this, I developed a C++11 linear algebra library used in conjunction with the OpenMP parallel computing library to empirically show a speed increase.

Preconditioning and the Conjugate Gradient Method in the Context of Solving PDEs÷is about the interplay between modeling, analysis, discretization, matrix computation, and model reduction. The authors link PDE analysis, functional analysis, and calculus of variations with matrix iterative computation using Krylov subspace methods and address the challenges that arise during formulation of the mathematical model through to efficient numerical solution of the algebraic problem. The book?s central concept, preconditioning of the conjugate gradient method, is traditionally developed algebraically using the preconditioned finite-dimensional algebraic system. In this text, however, preconditioning is connected to the PDE analysis, and the infinite-dimensional formulation of the conjugate gradient method and its discretization and preconditioning are linked together. This text challenges commonly held views, addresses widespread misunderstandings, and formulates thought-provoking open questions for further research.÷

The problem of minimizing the peak sidelobe level of a planar array of dipoles using phase-only synthesis is investigated. A hybrid nonlinear function minimization scheme is developed using a Taylor series approximation and a conjugate gradient algorithm. A weighted average peak sidelobe level function is introduced to yield a more stable numerical procedure. The peak sidelobe level is found to decrease in proportion to the logarithm of the aperture behavior.

Abstract: "In this report we describe how parallelism can be achieved in the conjugate gradient algorithm for solving linear equations. Both full and sparse version codes are implemented and respective speed up for both cases are discussed."

This book details algorithms for large-scale unconstrained and bound constrained optimization. It shows optimization techniques from a conjugate gradient algorithm perspective as well as methods of shortest residuals, which have been developed by the author.

The Lanczos and conjugate gradient (CG) algorithms are fascinating numerical algorithms. This book presents the most comprehensive discussion to date of the use of these methods for computing eigenvalues and solving linear systems in both exact and floating point arithmetic. The author synthesizes the research done over the past 30 years, describing and explaining the "average" behavior of these methods and providing new insight into their properties in finite precision. Many examples are given that show significant results obtained by researchers in the field. The author emphasizes how both algorithms can be used efficiently in finite precision arithmetic, regardless of the growth of rounding errors that occurs. He details the mathematical properties of both algorithms and demonstrates how the CG algorithm is derived from the Lanczos algorithm. Loss of orthogonality involved with using the Lanczos algorithm, ways to improve the maximum attainable accuracy of CG computations, and what modifications need to be made when the CG method is used with a preconditioner are addressed.

This book provides an introduction to probabilistic inductive logic programming. It places emphasis on the methods based on logic programming principles and covers formalisms and systems, implementations and applications, as well as theory.

While BP is inefficient on these ravine phenomena, it is shown that SCG handles them effectively."

Sparse Matrix Computations is a collection of papers presented at the 1975 Symposium by the same title, held at Argonne National Laboratory. This book is composed of six parts encompassing 27 chapters that contain contributions in several areas of matrix computations and some of the most potential research in numerical linear algebra. The papers are organized into general categories that deal, respectively, with sparse elimination, sparse eigenvalue calculations, optimization, mathematical software for sparse matrix computations, partial differential equations, and applications involving sparse matrix technology. This text presents research on applied numerical analysis but with considerable influence from computer science. In particular, most of the papers deal with the design, analysis, implementation, and application of computer algorithms. Such an emphasis includes the establishment of space and time complexity bounds and to understand the algorithms and the computing environment. This book will prove useful to mathematicians and computer scientists.

The concept of "reformulation" has long been playing an important role in mathematical programming. A classical example is the penalization technique in constrained optimization that transforms the constraints into the objective function via a penalty function thereby reformulating a constrained problem as an equivalent or approximately equivalent unconstrained problem. More recent trends consist of the reformulation of various mathematical programming prob lems, including variational inequalities and complementarity problems, into equivalent systems of possibly nonsmooth, piecewise smooth or semismooth nonlinear equations, or equivalent unconstrained optimization problems that are usually differentiable, but in general not twice differentiable. Because of the recent advent of various tools in nonsmooth analysis, the reformulation approach has become increasingly profound and diversified. In view of growing interests in this active field, we planned to organize a cluster of sessions entitled "Reformulation - Nonsmooth, Piecewise Smooth, Semismooth and Smoothing Methods" in the 16th International Symposium on Mathematical Programming (ismp97) held at Lausanne EPFL, Switzerland on August 24-29, 1997. Responding to our invitation, thirty-eight people agreed to give a talk within the cluster, which enabled us to organize thirteen sessions in total. We think that it was one of the largest and most exciting clusters in the symposium. Thanks to the earnest support by the speakers and the chairpersons, the sessions attracted much attention of the participants and were filled with great enthusiasm of the audience.

In this text, we introduce the basic concepts for the numerical modelling of partial differential equations. We consider the classical elliptic, parabolic and hyperbolic linear equations, but also the diffusion, transport, and Navier-Stokes equations, as well as equations representing conservation laws, saddle-point problems and optimal control problems. Furthermore, we provide numerous physical examples which underline such equations. We then analyze numerical solution methods based on finite elements, finite differences, finite volumes, spectral methods and domain decomposition methods, and reduced basis methods. In particular, we discuss the algorithmic and computer implementation aspects and provide a number of easy-to-use programs. The text does not require any previous advanced mathematical knowledge of partial differential equations: the absolutely essential concepts are reported in a preliminary chapter. It is therefore suitable for students of bachelor and master courses in scientific disciplines, and recommendable to those researchers in the academic and extra-academic domain who want to approach this interesting branch of applied mathematics.

Proceedings of the AMS-IMS-SIAM Summer Research Conference held at the University of Washington, July 1995.

The most comprehensive and up-to-date discussion available of the Lanczos and CG methods for computing eigenvalues and solving linear systems.

Here, we present an iterative algorithm for computing an invariant subspace associated with the algebraically smallest eigenvalues of a large sparse or structured Hermitian matrix A. We are interested in the case in which the dimension of the invariant subspace is large (e.g., over several hundreds or thousands) even though it may still be small relative to the dimension of A. These problems arise from, for example, density functional theory (DFT) based electronic structure calculations for complex materials. The key feature of our algorithm is that it performs fewer Rayleigh-Ritz calculations compared to existing algorithms such as the locally optimal block preconditioned conjugate gradient or the Davidson algorithm. It is a block algorithm, and hence can take advantage of efficient BLAS3 operations and be implemented with multiple levels of concurrency. We discuss a number of practical issues that must be addressed in order to implement the algorithm efficiently on a high performance computer.

The conjugate gradient method is a powerful tool for the iterative solution of self-adjoint operator equations in Hilbert space.This volume summarizes and extends the developments of the past decade concerning the applicability of the conjugate gradient method (and some of its variants) to ill posed problems and their regularization. Such problems occur in applications from almost all natural and technical sciences, including astronomical and geophysical imaging, signal analysis, computerized tomography, inverse heat transfer problems, and many more This Research Note presents a unifying analysis of an entire family of conjugate gradient type methods. Most of the results are as yet unpublished, or obscured in the Russian literature. Beginning with the original results by Nemirovskii and others for minimal residual type methods, equally sharp convergence results are then derived with a different technique for the classical Hestenes-Stiefel algorithm. In the final chapter some of these results are extended to selfadjoint indefinite operator equations. The main tool for the analysis is the connection of conjugate gradient type methods to real orthogonal polynomials, and elementary properties of these polynomials. These prerequisites are provided in a first chapter. Applications to image reconstruction and inverse heat transfer problems are pointed out, and exemplarily numerical results are shown for these applications.

Mathematics of Computing -- Numerical Analysis.

A comprehensive and updated overview of the theory, algorithms and applications of for electromagnetic inverse scattering problems Offers the recent and most important advances in inverse scattering grounded in fundamental theory, algorithms and practical engineering applications Covers the latest, most relevant inverse scattering techniques like signal subspace methods, time reversal, linear sampling, qualitative methods, compressive sensing, and noniterative methods Emphasizes theory, mathematical derivation and physical insights of various inverse scattering problems Written by a leading expert in the field