Crank Nicolson Solution To The Heat Equation

Mathematical models of various natural processes are described by differential equations, systems of partial differential equations and integral equations. In most cases, the exact solution to such problems cannot be determined; therefore, one has to use grid methods to calculate an approximate solution using high-performance computing systems. These methods include the finite element method, the finite difference method, the finite volume method and combined methods. In this Special Issue, we bring to your attention works on theoretical studies of grid methods for approximation, stability and convergence, as well as the results of numerical experiments confirming the effectiveness of the developed methods. Of particular interest are new methods for solving boundary value problems with singularities, the complex geometry of the domain boundary and nonlinear equations. A part of the articles is devoted to the analysis of numerical methods developed for calculating mathematical models in various fields of applied science and engineering applications. As a rule, the ideas of symmetry are present in the design schemes and make the process harmonious and efficient.

Matthäus Jäger examines the simulation of liquid-gas flow in fuel tank systems and its application to sloshing problems. The author focuses at first on the physical model and the assumptions necessary to derive the respective partial differential equations. The second step involves the cell-centered finite volume method and its application to fluid dynamic problems with free surfaces using a volume of fluid approach. Finally, the application of the method for different use cases is presented followed by an introduction to the methodology for the interpretation of the results achieved.

The early exercise opportunity of an American option makes it challenging to price and an array of approaches have been proposed in the vast literature on this topic. In The Numerical Solution of the American Option Pricing Problem, Carl Chiarella, Boda Kang and Gunter Meyer focus on two numerical approaches that have proved useful for finding all prices, hedge ratios and early exercise boundaries of an American option. One is a finite difference approach which is based on the numerical solution of the partial differential equations with the free boundary problem arising in American option pricing, including the method of lines, the component wise splitting and the finite difference with PSOR. The other approach is the integral transform approach which includes Fourier or Fourier Cosine transforms. Written in a concise and systematic manner, Chiarella, Kang and Meyer explain and demonstrate the advantages and limitations of each of them based on their and their co-workers'' experiences with these approaches over the years. Contents: Introduction; The Merton and Heston Model for a Call; American Call Options under Jump-Diffusion Processes; American Option Prices under Stochastic Volatility and Jump-Diffusion Dynamics OCo The Transform Approach; Representation and Numerical Approximation of American Option Prices under Heston; Fourier Cosine Expansion Approach; A Numerical Approach to Pricing American Call Options under SVJD; Conclusion; Bibliography; Index; About the Authors. Readership: Post-graduates/ Researchers in finance and applied mathematics with interest in numerical methods for American option pricing; mathematicians/physicists doing applied research in option pricing. Key Features: Complete discussion of different numerical methods for American options; Able to handle stochastic volatility and/or jump diffusion dynamics; Able to produce hedge ratios efficiently and accurately"

This book is open access under a CC BY 4.0 license. This easy-to-read book introduces the basics of solving partial differential equations by means of finite difference methods. Unlike many of the traditional academic works on the topic, this book was written for practitioners.

Accordingly, it especially addresses: the construction of finite difference schemes, formulation and implementation of algorithms, verification of implementations, analyses of physical behavior as implied by the numerical solutions, and how to apply the methods and software to solve problems in the fields of physics and biology.

This book is intended to determine the stability of one space dimension diffusion equation. A Matlab code of finite difference methods with increment of time-space was used in which the behaviour of the errors was observed from the graphs. The explicit scheme was stable with Dirichlet boundary condition when considering space for r less than or equal to 0.5. It was observed that as the gradient alpha of temperature decreases with derivative boundary conditions, the interval of r for the explicit scheme stet stable decreases from the values r less than or equal to 0.5 corresponding to Dirichlet boundary conditions. When the term with coefficient gamma is added to the PDE, explicit scheme becomes stable depending to the value of gamma. The Crank-Nicolson and semi-analytic schemes were stable with both Dirichlet boundary conditions and derivative boundary conditions for all r. It was observed that the Crank-Nicolson scheme was accurate than explicit scheme. The semi-analytic method has only one source of error, the space discretization also it is able to solve for a vector of time simultaneously. But with sufficient small r all three methods were performed well.

This textbook provides engineers with the capability, tools and confidence to solve real-world heat transfer problems. It includes many advanced topics, such as Bessel functions, Laplace transforms, separation of variables, Duhamel's theorem and complex combination, as well as high order explicit and implicit numerical integration algorithms. These analytical and numerical solution methods are applied to topics not considered in most textbooks. Examples include heat exchangers involving fluids with varying specific heats or phase changes; heat exchangers in which axial conduction is a concern; and regenerators. Derivations of important results are presented completely, without skipping steps, which reduces student frustration and improves readability and retention. The examples are not trivial 'textbook' exercises; they are rather complex and timely real-world problems that are inherently interesting. This book integrates the computational software packages Maple, MATLAB®, FEHT and Engineering Equation Solver (EES) directly with the heat transfer material.

Computational Techniques for Differential Equations

The transient heat conduction equation is solved for inhomogeneous media using the Explicit, Pure-Implicit, Crank-Nicolson and Douglas finite-difference methods, and the numerical solutions are investigated with respect to accuracy and stability. The inherent discontinuity between the initial and boundary conditions is accounted for by mesh refinement. For the two versions of the problem for which the four numerical methods are investigated, all four methods are found to be of equivalent accuracy for small values of the Fourier Modulus. While the Pure-Implicit, Crank-Nicolson and Douglas methods are unconditionally stable, the Crank-Nicolson and Douglas methods are very inaccurate at large values of the Fourier Modulus due to oscillatory behavior. (Author).

Control of Crank-Nicolson Noise in the Numerical Solution of the Heat Conduction EquationConvergence of the Solution of a Modified Crank-Nicolson Difference Equation to the Solution of a Quasi-linear Parabolic Differential EquationComputational Techniques for Differential EquationsElsevier

International Series of Monographs in Pure and Applied Mathematics, Volume 54: Integration of Equations of Parabolic Type by the Method of Nets deals with solving parabolic partial differential equations using the method of nets. The first part of this volume focuses on the construction of net equations, with emphasis on the stability and accuracy of the approximating net equations. The method of nets or method of finite differences (used to define the corresponding numerical method in ordinary differential equations) is one of many different approximate methods of integration of partial differential equations. The other methods, and some based on newer equations, are described. By analyzing these newer methods, older and existing methods are evaluated. For example, the asymmetric net equations; the alternating method of using certain equations; and the method of mean arithmetic and multi-nodal symmetric method point out that when the accuracy needs to be high, the requirements for stability become more defined. The methods discussed are very theoretical and methodological. The second part of the book concerns the practical numerical solution of the equations posed in Part I. Emphasis is on the commonly used iterative methods that are programmable on computers. This book is suitable for statisticians and numerical analysts and is also recommended for scientists and engineers with general mathematical knowledge.

Procedures and results are given for the numerical solution of viscous compressible flow in a slender channel. The results are compared with an exact solution developed through similarity. Two methods, normal implicit method and Crank-Nicolson method, were attempted in the numerical solution. The Crank-Nicolson method was found to be superior to the normal implicit method. The normal implicit method did have the advantage of requiring less computation time since the momentum and energy equations were solved separately by two sets of (N-2) equations with (N-2) unknowns. The normal implicit method also made possible the use of a smaller transverse grid size.

The two dimensional heat diffusion equation with Dirichlet boundary conditions was solved using the fully explicit, fully implicit, Crank-Nicolson implicit, and Peaceman-Rachford alternating direction implicit (ADI) methods. Comparisons of accuracy and time requirements were made. The possibility that the ADI method has stable oscillatory solutions with large time steps was investigated. Results of computations revealed that the ADI has stable oscillations for large time steps, in some cases producing large enough errors to render the solution unusable. Time steps greater than twice the square of the mesh spacing divided by the thermal diffusivity must be used with care. For small time steps, the Crank-Nicolson and ADI methods were the most accurate, and the ADI was the fastest method. The fully implicit method was the most accurate at large time steps, but the ADI, with a smaller time step to reduce the oscillatory error, was still the fastest method to reach a solution with the desired degree of accuracy. Additional keywords: theses; numerical analysis; finite difference theory; conduction (heat transfer); partial differential equations; stability.(Author).

Definitive Treatment of the Numerical Simulation of Bioheat Transfer and Fluid Flow Motivated by the upwelling of current interest in subjects critical to human health, Advances in Numerical Heat Transfer, Volume 3 presents the latest information on bioheat and biofluid flow. Like its predecessors, this volume assembles a team of renowned international researchers who cover both fundamentals and applications. It explores ingenious modeling techniques and innovative numerical simulation for solving problems in biomedical engineering. The text begins with the modeling of thermal transport by perfusion within the framework of the porous-media theory. It goes on to review other perfusion models, different forms of the bioheat equation for several thermal therapies, and thermal transport in individual blood vessels. The book then describes thermal methods of tumor detection and treatment as well as issues of blood heating and cooling during lengthy surgeries. It also discusses how the enhancement of heat conduction in tumor tissue by intruded nanoparticles improves the efficacy of thermal destruction of the tumor. The final chapters focus on whole-body thermal models, issues concerning the thermal treatment of cancer, and a case study on the thermal ablation of an enlarged prostate.

This postgraduate text describes methods which can be used to solve physical and chemical problems on a digital computer. The methods are described on simple, physical problems with which the student is familiar, and then extended to more complex ones. Emphasis is placed on the use of discrete grid points, the representation of derivatives by finite difference ratios, and the consequent replacement of the differential equations by a set of finite difference equations. Efficient methods for the solution of the resulting set of equations are given, and five solution algorithms are presented in the book.

Heat transfer analysis is a problem of major significance in a vast range of industrial applications. These extend over the fields of mechanical engineering, aeronautical engineering, chemical engineering and numerous applications in civil and electrical engineering. If one considers the heat conduction equation alone the number of practical problems amenable to solution is extensive. Expansion of the work to include features such as phase change, coupled heat and mass transfer, and thermal stress analysis provides the engineer with the capability to address a further series of key engineering problems. The complexity of practical problems is such that closed form solutions are not generally possible. The use of numerical techniques to solve such problems is therefore considered essential, and this book presents the use of the powerful finite element method in heat transfer analysis. Starting with the fundamental general heat conduction equation, the book moves on to consider the solution of linear steady state heat conduction problems, transient analyses and non-linear examples. Problems of melting and solidification are then considered at length followed by a chapter on convection. The application of heat and mass transfer to drying problems and the calculation of both thermal and shrinkage stresses conclude the book. Numerical examples are used to illustrate the basic concepts introduced. This book is the outcome of the teaching and research experience of the authors over a period of more than 20 years.

First Published in 2018. Routledge is an imprint of Taylor & Francis, an Informa company.

Nonlinear Partial Differential Equations (PDEs) have become increasingly important in the description of physical phenomena. Unlike Ordinary Differential Equations, PDEs can be used to effectively model multidimensional systems. The methods put forward in Discrete Variational Derivative Method concentrate on a new class of "structure-preserving num

Emphasizing the finite difference approach for solving differential equations, the second edition of Numerical Methods for

Engineers and Scientists presents a methodology for systematically constructing individual computer programs. Providing easy access to accurate solutions to complex scientific and engineering problems, each chapter begins with objectives, a discussion of a representative application, and an outline of special features, summing up with a list of tasks students should be able to complete after reading the chapter- perfect for use as a study guide or for review. The AIAA Journal calls the book "...a good, solid instructional text on the basic tools of numerical analysis."

The early exercise opportunity of an American option makes it challenging to price and an array of approaches have been proposed in the vast literature on this topic. In The Numerical Solution of the American Option Pricing Problem, Carl Chiarella, Boda Kang and Gunter Meyer focus on two numerical approaches that have proved useful for finding all prices, hedge ratios and early exercise boundaries of an American option. One is a finite difference approach which is based on the numerical solution of the partial differential equations with the free boundary problem arising in American option pricing, including the method of lines, the component wise splitting and the finite difference with PSOR. The other approach is the integral transform approach which includes Fourier or Fourier Cosine transforms. Written in a concise and systematic manner, Chiarella, Kang and Meyer explain and demonstrate the advantages and limitations of each of them based on their and their co-workers' experiences with these approaches over the years. Contents:IntroductionThe Merton and Heston Model for a CallAmerican Call Options under Jump-Diffusion ProcessesAmerican Option Prices under Stochastic Volatility and Jump-Diffusion Dynamics — The Transform ApproachRepresentation and Numerical Approximation of American Option Prices under HestonFourier Cosine Expansion ApproachA Numerical Approach to Pricing American Call Options under SVJDConclusionBibliographyIndexAbout the Authors Readership: Post-graduates/ Researchers in finance and applied mathematics with interest in numerical methods for American option pricing; mathematicians/physicists doing applied research in option pricing Keywords:American Option;Early Exercise;Method of Lines;Finite Difference Approach;Integral Transform Approach;Numerical MethodsKey Features:Complete discussion of different numerical methods for American optionsAble to handle stochastic volatility and/or jump diffusion dynamicsAble to produce hedge ratios efficiently and accurately

In a previous paper devoted to the numerical solution of the Stefan problem, the author has proposed a numerical scheme to solve the heat equation on a variable mesh; this scheme is a generalization of the classical Crank-Nicolson scheme since it is identical to the Crank-Nicolson scheme in the particular case of a fixed mesh. Numerical experiments have been performed in one and two space-dimensions, but no mathematical results had been proved. In the present paper, the stability and convergence of the scheme and established together with an error estimate. (Author).

The purpose of this two-volume textbook is to provide students of engineer ing, science and applied mathematics with

the specific techniques, and the framework to develop skill in using them, that have proven effective in the various branches of computational fluid dynamics (CFD). Volume 1 de scribes both fundamental and general techniques that are relevant to all branches of fluid flow. Volume 2 provides specific techniques, applicable to the different categories of engineering flow behaviour, many of which are also appropriate to convective heat transfer. An underlying theme of the text ist that the competing formulations which are suitable for computational fluid dynamics, e.g. the finite differ ence, finite element, finite volume and spectral methods, are closely related and can be interpreted as part of a unified structure. Classroom experience indicates that this approach assists, considerably, the student in acquiring a deeper understanding of the strengths and weaknesses of the alternative computational methods. Through the provision of 24 computer programs and associated exam ples and problems, the present text is also suitable for established research workers and practitioners who wish to acquire computational skills without the benefit of formal instruction. The text includes the most up-to-date techniques and is supported by more than 300 figures and 500 references.
Implementing Models of Financial Derivatives is a comprehensive treatment of advanced implementation techniques in VBA for models of financial derivatives. Aimed at readers who are already familiar with the basics of VBA it emphasizes a fully object oriented approach to valuation applications, chiefly in the context of Monte Carlo simulation but also more broadly for lattice and PDE methods. Its unique approach to valuation, emphasizing effective implementation from both the numerical and the computational perspectives makes it an invaluable resource. The book comes with a library of almost a hundred Excel spreadsheets containing implementations of all the methods and models it investigates, including a large number of useful utility procedures. Exercises structured around four application streams supplement the exposition in each chapter, taking the reader from basic procedural level programming up to high level object oriented implementations. Written in eight parts, parts 1-4 emphasize application design in VBA, focused around the development of a plain Monte Carlo application. Part 5 assesses the performance of VBA for this application, and the final 3 emphasize the implementation of a fast and accurate Monte Carlo method for option valuation. Key topics include: ?Fully polymorphic factories in VBA; ?Polymorphic input and output using the TextStream and FileSystemObject objects; ?Valuing a book of options; ?Detailed assessment of the performance of VBA data structures; ?Theory, implementation, and comparison of the main Monte Carlo variance reduction methods; ?Assessment of discretization methods and their application to option valuation in models like CIR and Heston; ?Fast valuation of Bermudan options by Monte Carlo. Fundamental theory and implementations of lattice and PDE methods are presented in appendices and developed through the book in the exercise streams. Spanning the two worlds of academic theory and industrial practice, this book is not only suitable as a classroom text in VBA, in simulation methods, and as an introduction to object oriented design, it

is also a reference for model implementers and quants working alongside derivatives groups. Its implementations are a valuable resource for students, teachers and developers alike. Note: CD-ROM/DVD and other supplementary materials are not included as part of eBook file.

Numerical Methods for Partial Differential Equations: Finite Difference and Finite Volume Methods focuses on two popular deterministic methods for solving partial differential equations (PDEs), namely finite difference and finite volume methods. The solution of PDEs can be very challenging, depending on the type of equation, the number of independent variables, the boundary, and initial conditions, and other factors. These two methods have been traditionally used to solve problems involving fluid flow. For practical reasons, the finite element method, used more often for solving problems in solid mechanics, and covered extensively in various other texts, has been excluded. The book is intended for beginning graduate students and early career professionals, although advanced undergraduate students may find it equally useful. The material is meant to serve as a prerequisite for students who might go on to take additional courses in computational mechanics, computational fluid dynamics, or computational electromagnetics. The notations, language, and technical jargon used in the book can be easily understood by scientists and engineers who may not have had graduate-level applied mathematics or computer science courses. Presents one of the few available resources that comprehensively describes and demonstrates the finite volume method for unstructured mesh used frequently by practicing code developers in industry Includes step-by-step algorithms and code snippets in each chapter that enables the reader to make the transition from equations on the page to working codes Includes 51 worked out examples that comprehensively demonstrate important mathematical steps, algorithms, and coding practices required to numerically solve PDEs, as well as how to interpret the results from both physical and mathematic perspectives

Basic option theory - Numerical methods - Further option theory - Interest rate derivative products.

This complementary text provides detailed solutions for the problems that appear in Chapters 2 to 18 of Computational Techniques for Fluid Dynamics (CTFD), Second Edition. Consequently there is no Chapter 1 in this solutions manual. The solutions are indicated in enough detail for the serious reader to have little difficulty in completing any intermediate steps. Many of the problems require the reader to write a computer program to obtain the solution. Tabulated data, from computer output, are included where appropriate and coding enhancements to the programs provided in CTFD are indicated in the solutions. In some instances completely new programs have been written and the listing forms part of the solution. All of the program modifications, new programs and input/output files are available on an IBM compatible floppy direct from C.A.J. Fletcher. Many of the problems are substantial enough to be considered mini-projects and the discussion is aimed as much at encouraging the reader to explore ex tensions and what-if scenarios leading to further dcvelopment as at providing neatly packaged solutions. Indeed, in order to givc the reader a better intro duction to CFD reality, not all the problems do have a "happy ending". Some suggested extensions fail; but the reasons for the failure are illuminating.

This text provides a very simple, initial introduction to the complete scientific computing pipeline: models, discretization, algorithms, programming, verification, and visualization. The pedagogical strategy is to use one case study – an ordinary differential equation describing exponential decay processes – to illustrate fundamental concepts in mathematics and computer science. The book is easy to read and only requires a command of one-variable calculus and some very basic knowledge about computer programming. Contrary to similar texts on numerical methods and programming, this text has a much stronger focus on implementation and teaches testing and software engineering in particular.

This book focuses on heat and mass transfer, fluid flow, chemical reaction, and other related processes that occur in engineering equipment, the natural environment, and living organisms. Using simple algebra and elementary calculus, the author develops numerical methods for predicting these processes mainly based on physical considerations. Through this approach, readers will develop a deeper understanding of the underlying physical aspects of heat transfer and fluid flow as well as improve their ability to analyze and interpret computed results.

This is a comparison study of the abilities of the eigenvalue method as a numerical method in solving the transient heat conduction equation. The eigenvalue method was compared to five other numerical methods; Runge-Kutta, Gears, extrapolation, fully implicit, and Crank-Nicolson. These methods were used to solved three physical problems. The first is a two dimensional slap which takes advantage of the symmetry of the problem. The second is a the same slap problem without taking advantage of the symmetry. And the third is a cylindrical problem taking full advantage of symmetry. The scope of the study is to see which methods take less computer time while maintaining sufficient accuracy. The time it takes the computer to totally execute the program was used as the time comparison basis. The accuracy is a comparison of the exact solution to the numerical solution. a root mean square average off all the grid points per time step is used. The results of the study were surprising. The accuracy of the eigenvalue method is not any better than that of the Crank-Nicolson method. The computer times show that the eigenvalue is not the fastest for short transient times. A long transient problem with nonlinear terms was not used in this study.

What makes this book stand out from the competition is that it is more computational. Once done with both volumes, readers will have the tools to attack a wider variety of problems than those worked out in the competitors' books. The author stresses the use of technology throughout the text, allowing students to utilize it as much as possible.

Theory, methods and software for elliptic (steady-state) and parabolic (diffusion) partial differential equations, plus linear algebra and error estimators.

the conference participants. We also thank M. Koleva for the help in putting together the book.

This more-of-physics, less-of-math, insightful and comprehensive book simplifies computational fluid dynamics for readers with little knowledge or experience in heat transfer, fluid dynamics or numerical methods. The novelty of this book lies in the simplification of the level of mathematics in CFD by presenting physical law (instead of the traditional differential equations) and discrete (independent of continuous) math-based algebraic formulations. Another distinguishing feature of this book is that it effectively links theory with computer program (code). This is done with pictorial as well as detailed explanations of implementation of the numerical methodology. It also includes pedagogical aspects such as end-of-chapter problems and carefully designed examples to augment learning in CFD code-development, application and analysis. This book is a valuable resource for students in the fields of mechanical, chemical or aeronautical engineering.

The world of quantitative finance (QF) is one of the fastest growing areas of research and its practical applications to derivatives pricing problem. Since the discovery of the famous Black-Scholes equation in the 1970's we have seen a surge in the number of models for a wide range of products such as plain and exotic options, interest rate derivatives, real options and many others. Gone are the days when it was possible to price these derivatives analytically. For most problems we must resort to some kind of approximate method. In this book we employ partial differential equations (PDE) to describe a range of one-factor and multi-factor derivatives products such as plain European and American options, multi-asset options, Asian options, interest rate options and real options. PDE techniques allow us to create a framework for modeling complex and interesting derivatives products. Having defined the PDE problem we then approximate it using the Finite Difference Method (FDM). This method has been used for many application areas such as fluid dynamics, heat transfer, semiconductor simulation and astrophysics, to name just a few. In this book we apply the same techniques to pricing real-life derivative products. We use both traditional (or well-known) methods as well as a number of advanced schemes that are making their way into the QF literature: Crank-Nicolson, exponentially fitted and higher-order schemes for one-factor and multi-factor options Early exercise features and approximation using front-fixing, penalty and variational methods Modelling stochastic volatility models using Splitting methods Critique of ADI and Crank-Nicolson schemes; when they work and when they don't work Modelling jumps using Partial Integro Differential Equations (PIDE) Free and moving boundary value problems in QF Included with the book is a CD containing information on how to set up FDM algorithms, how to map these algorithms to C++ as well as several working programs for one-factor and two-factor models. We also provide source code so that you can customize the applications to suit your own needs.

Finite Difference Methods in Heat Transfer presents a clear, step-by-step delineation of finite difference methods for solving engineering problems governed by ordinary and partial differential equations, with emphasis on heat transfer applications. The finite difference techniques presented apply to the numerical solution of problems governed by similar differential equations encountered in many other fields. Fundamental concepts are introduced in an easy-to-follow manner. Representative examples illustrate the application of a variety of powerful and widely used finite difference techniques. The physical situations considered include the steady state and transient heat conduction, phase-change involving melting and solidification, steady and transient forced convection inside ducts, free convection over a flat plate, hyperbolic heat conduction, nonlinear diffusion, numerical grid generation techniques, and hybrid numerical-analytic solutions.

Substantially revised, this authoritative study covers the standard finite difference methods of parabolic, hyperbolic, and elliptic equations, and includes the concomitant theoretical work on consistency, stability, and convergence. The new edition includes revised and greatly expanded sections on stability based on the Lax-Richtmeyer definition, the application of Pade approximants to systems of ordinary differential equations for parabolic and hyperbolic equations, and a considerably improved presentation of iterative methods. A fast-paced introduction to numerical methods, this will be a useful volume for students of mathematics and engineering, and for postgraduates and professionals who need a clear, concise grounding in this discipline.